General probability weighted moments for the three-parameter Weibull Distribution and their application in *S-N* curves modelling

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Abstract

Modelling the statistical fatigue behavior, based on fatigue test results is still a challenge for researchers and engineers. The model proposed by Castillo *et al.* is one alternative to describe this phenomenon, which is based on a three-parameter Weibull Distribution whose parameters should be estimated. There are several methods to determine these parameters, however there is no consensus about which is the most appropriate. In this article, a general formulation of the Probability Weighted Moments of the Weibull Distribution is presented in order to estimate the parameters mentioned above. Finally, an application with experimental data from concrete specimens and simulated data is presented.

Keywords:
Cycle loading, *S-N* curves, Weibull, Probability weighted moments, Estimation

1. Introduction

The presence of fatigue failures in metallic structures is a common technical problem, and the modelling of the fatigue life of these structures is still a research challenge in the materials science since the 19th century [1]. At that time, August Wöhler[1] recognized, that applying a single load, which is much lower than the static strength of a structure, does not damage it, but if this load is applied several times, it could induce a complete failure of the structure. The failure begins

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[1] 1819-1914. German engineer who made a systematic research of fatigue

Model	Wöhler curves
Basquin (1910)	$\log N = A - B \log \Delta\sigma; \quad \Delta\sigma \geq \Delta\sigma_\infty$
Stomeyer (1914)	$\log N = A - B \log(\Delta\sigma - \Delta\sigma_\infty)$
Bastenaire (1972)	$N = \frac{A}{\Delta\sigma - E} \exp[-C(\Delta\sigma - E)] - B$
Pascual & Meeker (1999)	$\log N = A - B \log(\Delta\sigma - \Delta\sigma_\infty)$
Kohout & Věchet (2001)	$\log\left(\frac{\Delta\sigma}{\Delta\sigma_\infty}\right) = \log\left(\frac{N+N_1}{N+N_2}\right)^b$

Table 1: Common models to represent the S-N curves.

with the occurrence and growth of micro-cracks which are basis for macro cracks leading to collapse.

With the stress based approach, several models have been proposed to represent the S-N curves [2], [3], [4], [5], [6], [7], [8], which play a crucial role for the structural design; some of them are listened in table 1. Most of them only consider physical arguments and empiric data, and unfortunately, they only represent an elementary geometric approach which offers a limited judgement of the experimental results. Moreover, from the statistical point of view, some models do not propose a cummulative distribution function, so that, they are not suitable to extrapolate the results into the high cycle fatigue region [9], [10], [11]. In other words, it is not possible to predict with a probability p the fatigue life of a structure under a significant lower stress value as the tested experiments.

Based on a Weibull Distribution, Castillo *et al.* [12] proposed a probabilistic methodology for predicting the number of load cycles leading to failure of structural details. In contrary the traditional methods, this proposal emphasizes the stochastic nature of fatigue by considering both the stress range and the lifetime (number of load cycles) as random variables [13], [14], ensures a dimensional consistency [15] and considers the influence the run-outs obtained during the experiments.

The methodology mentioned above demands the estimation of the three parameters of the Weibull Distribution. In this article, a general formulation of the Probability Weighted Moments (PWM) of the Weibull Distribution is presented and applied to estimate these parameters. The application is based on (a) Experimental data from concrete specimens. (b) Data from computer simulation.

2. Weibull model

The three-parameter Weibull Distribution denoted by $W(a,b,c)$ is a member of the family of extreme value distributions. The cumulative distribution function (CDF) (also called life distribution or failure distribution) of a random variable x which follows a $W(a,b,c)$ [16] is given by

$$F(x \mid a,b,c) = 1 - \exp\left[-\left(\frac{x-a}{b}\right)^{c}\right], \qquad x \geq a \tag{1}$$

where

$a \in \mathbb{R}$: Location parameter (minimum life)

$b > 0$: Scale parameter (characteristic life)

$c > 0$: Shape parameter (slope of $F(x \mid a,b,c)$)

In the case of fatigue modelling, Castillo *et al.* consider that the conditional CDF of the random variables lifetime and stress range are not independent and must satisfy a compatibility condition which leads to a functional equation [13]. The solution of this functional equation leads to the following probabilistic fatigue model for a constant stress range, which is based on a Weibull Distribution [14], where the random variable is $x = (\log N - B)(\log \Delta\sigma - C)$.

$$Q(N,\Delta\sigma) = 1 - \exp\left\{-\left[\frac{(\log N - B)(\log \Delta\sigma - C) - a}{b}\right]^{c}\right\} \tag{2}$$

where

$\Delta\sigma$: stress range during the test

N : number of load cycles up to failure during the test

B : threshold value of lifetime N

C : endurance limit for $\Delta\sigma$

The model given by Eq.(2) has two geometrical parameters B and C, and three Weibull parameters a, b and c, which should be estimated.

The estimation of these parameters is done in two steps. First, the geometrical parameters are determined and second the Weibull parameters.

3. Parameter estimation

Consider that n experimental data points from stress ranges and load cycles are given as follows

$$N_i = N_1, N_2, \dots N_n$$

$$\Delta\sigma_i = \Delta\sigma_1, \Delta\sigma_2, \dots \Delta\sigma_n$$

The geometrical parameters B, C and the mean[2] μ of $W(a,b,c)$ are estimated by solving the following non-linear optimization problem [12].

$$\min_{B,C,\mu\in\mathbb{R}} \quad \sum_{i=1}^{n} \left(\log N_i - B - \frac{\mu}{\log\Delta\sigma_i - C} \right)^2 \tag{3}$$

Within this paper, the PWMs introduced by Greenwood at all [17] and applied to the Extreme Value Distribution [18] are considered.

The PWMs of a random variable X with CDF F are the quantities

$$M_{p,r,s} = \int_0^1 \left[x(F)\right]^p F^r (1-F)^s \,\mathrm{d}F, \tag{4}$$

where $p,r,s \in \mathbb{N}$.

In the subsequent subsections, the deduction and properties of $M_{1,0,s}$, $M_{1,r,0}$, $M_{1,r,s}$ and $M_{p,r,s}$ from a Weibull Distribution are presented.

3.1. Moments $M_{1,0,s}$

The inverse function of Eq.(1) is given by

$$x(F) = a + b\left[-\log(1-F)\right]^{\frac{1}{c}}, \tag{5}$$

replacing $p = 1$ and $r = 0$ in Eq.(4) gives the following PWMs.

$$M_{1,0,s} = a\int_0^1 (1-F)^s \,\mathrm{d}F + b\int_0^1 \left[-\log(1-F)\right]^{\frac{1}{c}} (1-F)^s \,\mathrm{d}F. \tag{6}$$

[2] Theoretically the mean of $W(a,b,c)$ is given by $\mu = a + b\Gamma(1+1/c)$

The first integral of Eq.(6) is equal to $\frac{a}{s+1}$.

Substituting $u = -\log(1-F)$ in the second integral of Eq.(6) leads to

$$b\int_0^\infty u^{\frac{1}{c}} e^{-(s+1)u}\,\mathrm{d}u, \tag{7}$$

substituting $x = (s+1)u$ in Eq.(7) leads to

$$\frac{b}{(s+1)^{1+\frac{1}{c}}}\int_0^\infty x^{\frac{1}{c}} e^{-x}\,\mathrm{d}x = \frac{b}{(s+1)^{1+\frac{1}{c}}}\Gamma\left(1+\frac{1}{c}\right), \tag{8}$$

where, $\Gamma(z)$ is de Gamma function defined by

$$\Gamma(z) = \int_0^\infty x^{z-1} e^{-x}\,\mathrm{d}x, \qquad z > 0. \tag{9}$$

Therefore, the PWM $M_{1,0,s}$ for the three-parameters Weibull Distribution are given by

$$M_{1,0,s} = \frac{a}{s+1} + \frac{b}{(s+1)^{1+\frac{1}{c}}}\Gamma\left(1+\frac{1}{c}\right), \qquad c > 0. \tag{10}$$

Now, the first three PWMs are considered in order to estimate the Weibull parameters a, b and c.
Denoting $M_s = M_{1,0,s},\ s = 0, 1, 2$ and $\Gamma_c = \Gamma(1+\frac{1}{c})$ in Eq.(10) the following system of equations is obtained

$$M_0 = a + b\Gamma_c \tag{11}$$

$$M_1 = \frac{a}{2} + \frac{b}{2^{1+\frac{1}{c}}}\Gamma_c \tag{12}$$

$$M_2 = \frac{a}{3} + \frac{b}{3^{1+\frac{1}{c}}}\Gamma_c. \tag{13}$$

Combining Eq.(11) and (12) gives

$$2M_1 - M_0 = b\Gamma_c\left(2^{\frac{-1}{c}} - 1\right). \tag{14}$$

Combining Eq.(11) and (13) gives

$$3M_2 - M_0 = b\Gamma_c\left(3^{\frac{-1}{c}} - 1\right). \tag{15}$$

Then dividing Eq.(15) by Eq.(14), leads to an equation for the parameter c.

$$\frac{3M_2 - M_0}{2M_1 - M_0} = \frac{3^{\frac{-1}{c}} - 1}{2^{\frac{-1}{c}} - 1} \tag{16}$$

which should be solved by numerical methods.

From Eq.(14), the value of the parameter b is

$$b = \frac{2M_1 - M_0}{(2^{\frac{-1}{c}} - 1)\Gamma_c}. \tag{17}$$

From Eq.(11), the value of the parameter a is

$$a = M_0 - b\Gamma_c. \tag{18}$$

It is necessary to know the value of the PWMs M_0, M_1, M_2 to solve the equations (16)-(18). For this reason, their estimators which depend on the ordered experimental data are used.

Let $x_1 < x_2 < \ldots < x_n$ be the order sample of the experimental data points. Then the estimators of the first three PWMs M_s [16] are given by

$$\widehat{M_0} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{19}$$

$$\widehat{M_1} = \frac{1}{n(n-1)}\sum_{i=1}^{n-1}(n-i)x_i \tag{20}$$

$$\widehat{M_2} = \frac{1}{n(n-1)(n-2)}\sum_{i=1}^{n-2}(n-i)(n-i-1)x_i. \tag{21}$$

By substituting the estimators given by equations (19)-(21) in the equations (16)-(18) the value of the Weibull parameters can be determined.

3.2. Moments $M_{1,r,0}$

Replacing the Eq.(5) into Eq.(4) for $p = 1$ and $s = 0$ gives the following PWMs.

$$M_{1,r,0} = a\int_0^1 F^r \,\mathrm{d}F + b\int_0^1 \big[-\log(1-F)\big]^{\frac{1}{c}} F^r \,\mathrm{d}F. \tag{22}$$

The first integral of Eq.(22) is equal to $\frac{a}{r+1}$.

Substituting $u = -\log(1-F)$ in the second integral of Eq.(22) gives

$$b\int_0^\infty u^{\frac{1}{c}} \left(1-e^{-u}\right)^r e^{-u}\,\mathrm{d}u \tag{23}$$

now, applying the binomial theorem given by

$$(1-a)^n = \sum_{k=0}^{n} \binom{n}{k} a^k (-1)^k, \tag{24}$$

it can be proved that

$$(1-e^{-u})^r = \sum_{k=0}^{r} \binom{r}{k} e^{-uk} (-1)^k. \tag{25}$$

Then, the integral of Eq.(23) becomes

$$b\int_0^\infty \sum_{k=0}^{r} (-1)^k \binom{r}{k} u^{\frac{1}{c}} e^{-(k+1)u}\,\mathrm{d}u = b\sum_{k=0}^{r} (-1)^k \binom{r}{k} \int_0^\infty u^{\frac{1}{c}} e^{-(k+1)u}\,\mathrm{d}u. \tag{26}$$

Substituting $x = (k+1)u$ in the integral of Eq.(26) gives

$$\frac{1}{(k+1)^{1+\frac{1}{c}}} \int_0^\infty x^{\frac{1}{c}} e^{-x}\,\mathrm{d}x = \frac{1}{(k+1)^{1+\frac{1}{c}}} \Gamma\left(1+\frac{1}{c}\right). \tag{27}$$

Thus, the integral of Eq.(23) becomes

$$I_4 = b\Gamma\left(1+\frac{1}{c}\right) \sum_{k=0}^{r} \frac{(-1)^k \binom{r}{k}}{(k+1)^{1+\frac{1}{c}}}. \tag{28}$$

Finally, the PWM $M_{1,r,0}$ for the three-parameters Weibull Distribution are given by

$$M_{1,r,0} = \frac{a}{r+1} + b\Gamma\left(1+\frac{1}{c}\right)\sum_{k=0}^{r}\frac{(-1)^k\binom{r}{k}}{(k+1)^{1+\frac{1}{c}}}, \qquad c>0. \tag{29}$$

Similar to the previous case, consider the following first three PWMs.
Denoting $M_r = M_{1,r,0},\ r = 0, 1, 2$ and $\Gamma_c = \Gamma(1+\frac{1}{c})$ in Eq.(29) the following system of equations is obtained

$$M_0 = a + b\Gamma_c \tag{30}$$

$$M_1 = \frac{a}{2} + \left(1 - \frac{1}{2^{1+\frac{1}{c}}}\right)b\Gamma_c \tag{31}$$

$$M_2 = \frac{a}{3} + \left(1 - \frac{1}{2^{1+\frac{1}{c}}} + \frac{1}{3^{1+\frac{1}{c}}}\right)b\Gamma_c \tag{32}$$

Combining Eq.(30) and (31) leads to

$$2M_1 - M_0 = \left(1 - 2^{\frac{-1}{c}}\right)b\Gamma_c. \tag{33}$$

Combining Eq.(30) and (32) gives

$$3M_2 - M_0 = \left(2 - 3\cdot 2^{\frac{-1}{c}} + 3^{\frac{-1}{c}}\right)b\Gamma_c. \tag{34}$$

By dividing Eq.(34) by Eq.(33) an equation for the parameter c is obtained

$$\frac{3M_2 - M_0}{2M_1 - M_0} = \frac{2 - 3\cdot 2^{-\frac{1}{c}} + 3^{-\frac{1}{c}}}{1 - 2^{-\frac{1}{c}}}, \tag{35}$$

which should be solved by numerical methods.

From Eq.(33) the value of the parameter b is obtained

$$b = \frac{2M_1 - M_0}{(1 - 2^{-\frac{1}{c}})\Gamma_c}. \tag{36}$$

From Eq.(30) the value of the parameter a is given by

$$a = M_0 - b\Gamma_c. \tag{37}$$

Finally, it is necessary again to know the values of the PWMs M_0, M_1, M_2 to solve the equations (35)-(37). Thus, their estimators which depend on the ordered experimental data points are used.
Let $x_1 < x_2 < \ldots < x_n$ be the order sample of the experimental data points. Then, the estimators of the first three PWMs M_r [19], [20] are given by

$$\widehat{M_0} = \frac{1}{n}\sum_{i=1}^{n} x_i \tag{38}$$

$$\widehat{M_1} = \frac{1}{n(n-1)}\sum_{i=1}^{n}(i-1)x_i \tag{39}$$

$$\widehat{M_2} = \frac{1}{n(n-1)(n-2)}\sum_{i=1}^{n}(i-1)(i-2)x_i. \tag{40}$$

By substituting the estimators given by equations (38)-(40) in the equations (35)-(37) the value of the Weibull parameters can be determined.

3.3. Moments $M_{1,r,s}$

Replacing Eq.(5) in Eq.(4) for $p = 1$ gives the following quasi general equation of the PWMs.

$$M_{1,r,s} = a\int_0^1 F^r(1-F)^s \,\mathrm{d}F + b\int_0^1 \left[-\log(1-F)\right]^{\frac{1}{c}} F^r(1-F)^s \,\mathrm{d}F. \tag{41}$$

Applying the binomial theorem given by Eq.(24), the first integral of Eq.(41) becomes

$$a\int_0^1 F^r \sum_{k=0}^{s}\binom{s}{k}F^k(-1)^k \,\mathrm{d}F = a\sum_{k=0}^{s}\binom{s}{k}\frac{(-1)^k}{k+r+1}. \tag{42}$$

On the second integral of Eq.(41) making the substitution $u = -\log(1-F)$ and applying also the binomial theorem results in

$$
\begin{aligned}
& b\int_0^\infty u^{\frac{1}{c}}\left(1-e^{-u}\right)^r e^{-(s+1)u}\,\mathrm{d}u \\
& = b\int_0^\infty u^{\frac{1}{c}} e^{-(s+1)u}\sum_{k=0}^{r}\binom{r}{k} e^{-ku}(-1)^k \\
& = b\sum_{k=0}^{r}\binom{r}{k}(-1)^k\int_0^\infty u^{\frac{1}{c}} e^{-(k+s+1)u}\,\mathrm{d}u.
\end{aligned}
\tag{43}
$$

Substituting $x=(k+s+1)u$ in the integral of Eq.(43) gives

$$
\frac{1}{(k+s+1)^{1+\frac{1}{c}}}\int_0^\infty x^{\frac{1}{c}} e^{-x}\,\mathrm{d}x = \frac{1}{(k+s+1)^{1+\frac{1}{c}}}\Gamma\left(1+\frac{1}{c}\right),
\tag{44}
$$

then, Eq.(43) becomes

$$
b\Gamma\left(1+\frac{1}{c}\right)\sum_{k=0}^{r}\binom{r}{k}\frac{(-1)^k}{(k+s+1)^{1+\frac{1}{c}}}
\tag{45}
$$

Therefore, the PWM $M_{1,r,s}$ for the three-parameters Weibull Distribution are obtained

$$
\begin{aligned}
M_{1,r,s} = \;& a\sum_{k=0}^{s}\binom{s}{k}\frac{(-1)^k}{k+r+1} \\
+ \;& b\Gamma\left(1+\frac{1}{c}\right)\sum_{k=0}^{r}\binom{r}{k}\frac{(-1)^k}{(k+s+1)^{1+\frac{1}{c}}} \quad c>0.
\end{aligned}
\tag{46}
$$

3.4. Moments $M_{p,r,s}$

Replacing Eq.(5) into Eq.(4) gives a general equation of the PWMs.

$$
M_{p,r,s} = \int_0^1\left\{a+b\left[-\log(1-F)\right]^{\frac{1}{c}}\right\}^p F^r(1-F)^s\,\mathrm{d}F.
\tag{47}
$$

Making the substitution $u=-\log(1-F)$ into Eq.(47) results in

$$
M_{p,r,s} = \int_0^\infty\left[a+bu^{\frac{1}{c}}\right]^p(1-e^{-u})^r e^{-(s+1)u}\,\mathrm{d}u.
\tag{48}
$$

Applying the binomial theorem given by Eq.(24) in the first term of the integral of Eq.(48) leads to

$$M_{p,r,s} = \sum_{i=0}^{p} \binom{p}{i} a^{p-i} b^{i} \int_{0}^{\infty} u^{\frac{i}{c}} (1-e^{-u})^{r} e^{-(s+1)u} \, \mathrm{d}u. \tag{49}$$

Then, applying again the binomial theorem, the integral of Eq.(49) becomes

$$\begin{aligned} &\int_{0}^{\infty} u^{\frac{i}{c}} e^{-(s+1)u} \sum_{k=0}^{r} \binom{r}{k} e^{-ku} (-1)^{k} \\ &= \sum_{k=0}^{r} \binom{r}{k} (-1)^{k} \int_{0}^{\infty} u^{\frac{i}{c}} e^{-(k+s+1)u} \, \mathrm{d}u \\ &= \sum_{k=0}^{r} \binom{r}{k} (-1)^{k} \frac{\Gamma\left(1+\frac{i}{c}\right)}{(s+k+1)^{1+\frac{i}{c}}}. \end{aligned} \tag{50}$$

Finally, the general PWM $M_{p,r,s}$ for the three-parameters Weibull Distribution can be calculated from

$$M_{p,r,s} = \sum_{i=0}^{p} \binom{p}{i} a^{p-i} b^{i} \sum_{k=0}^{r} \binom{r}{k} (-1)^{k} \frac{\Gamma\left(1+\frac{i}{c}\right)}{(s+k+1)^{1+\frac{i}{c}}}, \quad c > 0. \tag{51}$$

4. Application and results

In this section the method of the PWMs is applied in order to estimate the three parameters of a Weibull Distribution. The data arise from two different sources. a) The experimental data reported by Holmen (1979). b) Simulated data which were generated with Matlab.

In every case, a comparison between the results is presented.

4.1. Experimental data from Holmen

These 75 data are described by Holmen [21]. A sample of 15 concrete specimens was tested at each of the five levels $\Delta\sigma_i = \{0.675, 0.75, 0.825, 0.90, 0.95\}$. Here $\Delta\sigma_i$ is the ratio S_{max}/S_f where S_{max} is the maximum applied stress and S_f is the stress leading to static failure. The lifetimes are measured as the number of cycles to failure divided by 1000. The data are shown in Table 2.

The values of the geometrical parameters are already given as $B = -20.783$ and $C = -1.10607$, which are taken from [12].

The S-N curves obtained from Castillo-Hadi method [19] in Figure 1 and from the PWM method in Figure 2 have a similar geometry, however the S-N percentiles in Figure 2 are more narrow than the percentiles in Figure 1.

Holmen Data					
$\Delta\sigma$	N				
0.95	0.257	0.217	0.206	0.203	0.143
	0.123	0.120	0.109	0.105	0.085
	0.083	0.076	0.074	0.072	0.037
0.90	1.129	0.680	0.540	0.509	0.457
	0.451	0.356	0.342	0.311	0.295
	0.257	0.252	0.226	0.216	0.201
0.825	5.598	5.560	4.820	4.110	3.847
	3.590	3.330	2.903	2.590	2.410
	2.400	1.492	1.460	1.258	1.246
0.750	67.340	50.090	48.420	36.350	27.940
	26.260	24.900	20.300	18.620	17.280
	16.190	15.580	12.600	9.930	6.710
0.675	11748	11748	3295	1459	1400
	1330	1250	1242	896	659
	486	367	340	280	103

Table 2: Holmen data. Fatigue results with concrete specimen.

4.2. Simulated data

In this subsection, 75 data points of a Weibull Distribution $W(18, 1.5, 3)$ obtained through a Matlab numerical simulation are considered. In order to compare easily the results of the PWM method, the simulation of the data took into consideration the same values of the geometrical parameters given by $B = -20.783$ and $C = -1.10607$ from Holmen's data [12]. The simulated data are shown in Table 3.

In this case the S-N curve based on the PWM method has a quite similar geometry to the simulated curve. Particularly, the estimation of the location parameter a and scale parameter b do not differ too much from the theoretical values.

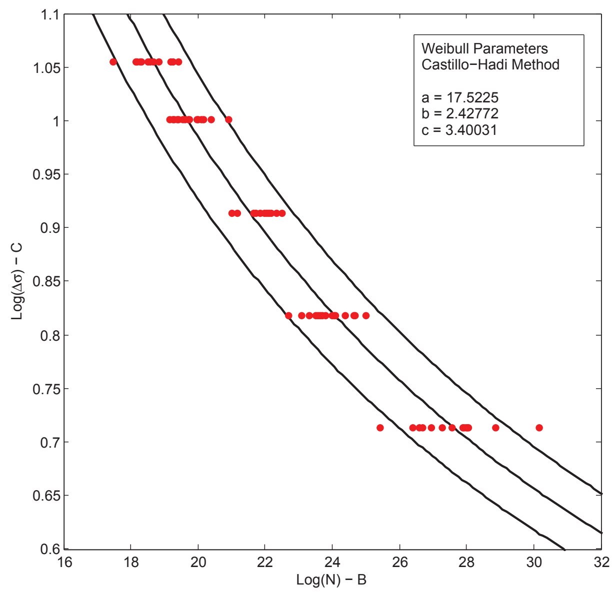

Figure 1: S-N curves from Holmen data from Table 2 based on the Castillo-Hadi estimators.

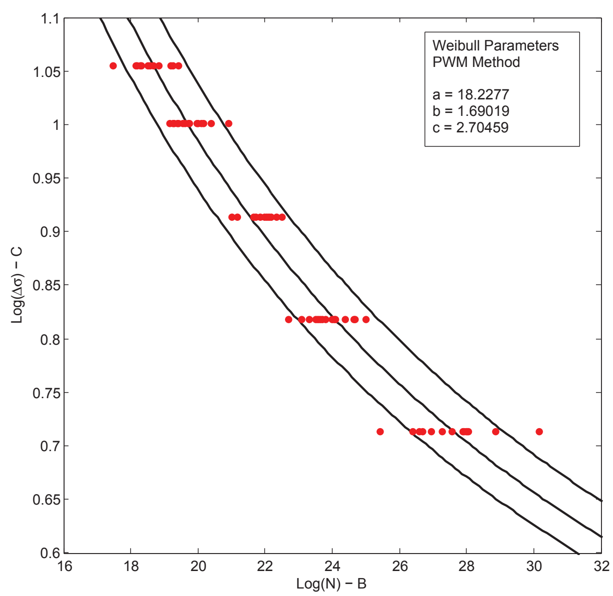

Figure 2: S-N curves from Holmen from Table 2 based on the PWMs estimators.

Simulated Data					
$\Delta\sigma$	N				
0.95	0.1122	0.0797	0.3518	0.0665	0.1154
	0.1085	0.0830	0.0578	0.0409	0.0865
	0.0623	0.1277	0.0812	0.1552	0.1293
0.90	0.2846	0.1638	0.6243	0.1893	0.5415
	0.1774	0.2203	0.1348	0.3122	0.1649
	0.1173	0.3521	0.2127	0.3212	0.2234
0.825	2.0085	0.6152	3.5572	2.6906	2.4286
	2.7607	1.4175	1.9787	1.8619	1.7452
	1.2023	1.1509	1.5627	1.1102	2.7119
0.750	79.5209	7.0670	21.5250	9.1130	34.7267
	84.1532	8.4761	53.6530	6.4612	27.5026
	33.3542	21.4731	43.2928	45.3180	24.9048
0.675	331.7648	407.5034	373.9940	226.5447	301.0640
	873.9784	2152.8246	540.3167	599.9790	3271.1257
	2232.2824	982.9569	3535.9388	1195.1812	522.2521

Table 3: Simulated data from a Weibull distribution $W(18, 1.5, 3)$

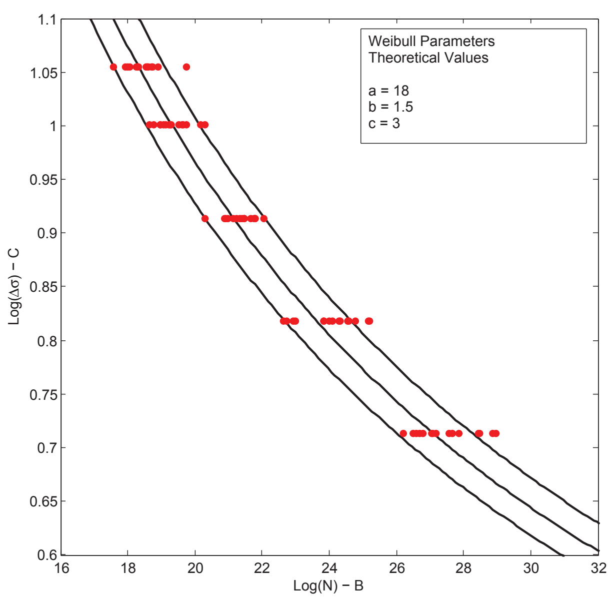

Figure 3: S-N curves from the simulated data from Table 3.

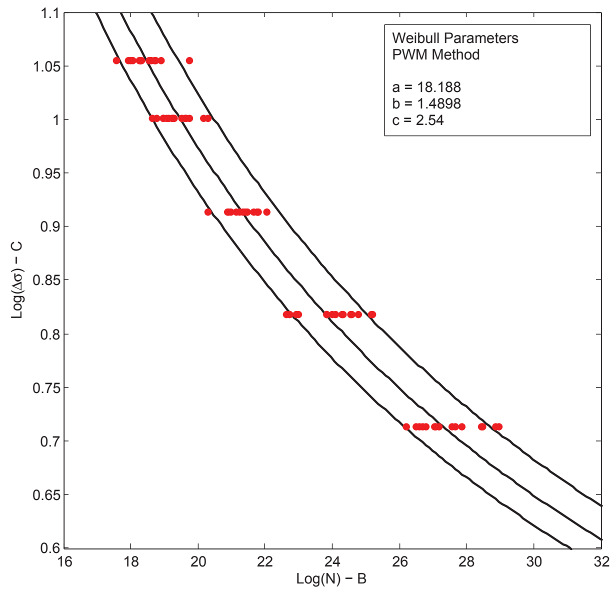

Figure 4: S-N curves from the simulated data from Table 3 based on the PWMs estimators.

5. Conclusions and future work

The PWM method offers a good alternative to estimate the three parameters of the Weibull Distribution. The obtained estimations do not differ significantly from the values of the simulated distribution, see Figures 3, 4.

The corresponding S-N curves of both applications are quite similar in the high cycle region. These curves describe properly the fatigue's properties such as the existence of endurance limit $\Delta\sigma_\infty$.

The endurance limit and the extrapolation of the S-N curves in the high cycle region are estimated in a better way than in the traditional linear regression model [6] which is currently used to represent the S-N curves and the fatigue data respectively. This linear regression do not allow to consider the existence of the endurance limit. In other words, the asymptotic behaviour obtained from the low stress values is neglected.

The Weibull model produces good results because of its statistical approach, which can be extrapolated if the experimental data points are abundant and come from the high cycles region as it is seen in Figures 1-4.

One of the problems regarding fatigue experiments on notched steel details is that the specimens and their tests are very expensive and take a very long time. This situation becomes more complicated if we want to apply the Weibull model, because it demands more data particularly from the high cycle region.

In the future in order to overcome this problem, the following situation is assumed.

Several run-outs are obtained from specimens which resisted the first fatigue test after a predetermined maximum number of applied load cycles.

Then, these resistant specimens are re-tested again with a higher stress value. If the specimen breaks, an additional value of failure is obtained. Otherwise, the specimen can be re-tested once again with a higher stress value.

Therefore, it would be interesting to establish if it is possible, to use the information obtained from the subsequent experiments in order to improve the estimation of the Weibull fatigue model. For this purpose it is mandatory to define two aspects: (a) A suitable definition of a damage accumulation function which considers (where pertinent): the influence of the coaxing effect, material properties, frequency of the test machine, etc. This function will allow taking into account the accumulated damage on a run-out for the subsequent tests. (b) A censoring sample model and the method to estimate the Weibull parameters based on it.

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References

[1] W. Schütz, Zur Geschichte der Schwingfestigkeit, Materialwissenschaft und Werkstofftechnik 24 (6) (1993) 203–232.

[2] F. Bastenaire, New method for the statistical evaluation of constant stress amplitude fatigue-test results, Probabilistic Aspects of Fatigue STP (511) (1972) 3–28.

[3] G. Marquis, M. Huther, A. Galtier, Guidance for the application of the best practice guide on statistical analysis of fatigue data, IIW-WG1-132-08.

[4] F. Pascual, W. Meeker, Estimating fatigue curves with the random fatigue-limit model, Technometrics 41 (4) (1999) 277–290.

[5] J. Schijve, Fatigue of Structures and Materials, Springer Verlag, 2009.

[6] Sedlacek, Hobbacher, Nussbaumer, Stötzel, 1^{st} draft of the background document prEN 1993-1-9, Eurocode 3: Design of steel structures.

[7] C. Stromeyer, The determination of fatigue limits under alternating stress conditions, Proceedings of the Royal Society of London 90 (620) (1914) 411–425.

[8] J. Kohout, S. Věchet, A new function for fatigue curves characterization and its multiple merits, International Journal of Fatigue 23 (2) (2001) 175–183.

[9] T. Lassen, P. Darcis, N. Recho, Fatigue behavior of welded joints part 1 - statistical methods for fatigue life prediction, Welding Journal (2005) 183–187.

[10] T. Lassen, N. Recho, Proposal for a more accurate physically based S-N curve for welded steel joints, International Journal of Fatigue 31 (1) (2009) 70–78.

[11] T. Lassen, N. Recho, Life prediction of welded steel joints - accuracy and shape of the S-N curves under constant and variable amplitude loading, Fatigue Design 2009.

[12] E. Castillo, A. Fernández-Canteli, A Unified Statistical Methodology for Modeling Fatigue Damage, Springer Verlag, 2009.

[13] E. Castillo, V. Esslinger, A. Fernández-Canteli, Statistical model for fatigue analysis of wires, strands and cables, in: IABSE Proceedings, Vol. 9, IABSE, IABSE Periodica, 1985.

[14] E. Castillo, J. Galambos, Lifetime regression models based on a functional equation of physical nature, Journal of Applied Probability 24 (1) (1987) 160–169.

[15] E. Buckingham, On physically similar systems; illustrations of the use of dimensional equations, Phys. Rev 4 (1914) 345–376.

[16] H. Rinne, The Weibull Distribution, Crc Pr Inc, 2008.

[17] J. Greenwood, J. Landwehr, N. Matalas, J. Wallis, Probability weighted moments: Definition and relation to parameters of several distributions expressable in inverse form, Water Resources Research 15 (05) (1979) 1049–1054.

[18] J. Hosking, J. Wallis, E. Wood, Estimation of the generalized extreme-value distribution by the method of probability-weighted moments, Technometrics 27 (3) (1985) 251–261.

[19] E. Castillo, A. S. Hadi, Parameter und quantile estimation for the generalized extreme-value distribution, Environmetrics 5 (1994) 417–432.

[20] D. J. Dupuis, Parameter and quantile estimation for the generalized extreme-value distribution: a second look, Environmetrics 10 (1999) 119–124.

[21] J. Holmen, Fatigue of concrete by constant and variable amplitude loading, Ph.D. thesis, University of Trondheim (1979).